

11 C

Ad Soyad

No

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11. Sınıf I. DÖNEM II. Matematik YAZILISI

1

 $i^2 = -1$ olmak üzere

$$-i^6 + i^7 - i^8 + i^9 - \dots - i^{100} + i^{101} = ?$$

10 puan

$$= \underbrace{-i^6 + i^7 - i^8 + i^9}_{=0} - \dots - \underbrace{i^{98} + i^{99} - i^{100} + i^{101}}_{=0}$$

$$= \underbrace{-i^2 + i^3 - i^0 + i^1}_{=0} - \dots - \underbrace{i^2 + i^3 - i^0 + i^1}_{=0} = 0$$

10 puan

2

$$z = 2 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) \rightarrow \text{Arg}(-i \cdot \bar{z}) = ?$$

$$z = 2 \cdot \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right)$$

$$z = -1 + \sqrt{3}i$$

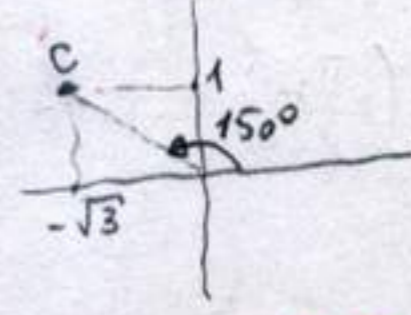
$$\bar{z} = -1 - \sqrt{3}i$$

$$-i \cdot \bar{z} = -i(-1 - \sqrt{3}i)$$

$$= i + \sqrt{3}i \cdot i$$

$$= -\sqrt{3} + i = c$$

$$\text{Arg}(-i \cdot \bar{z}) = 150^\circ$$



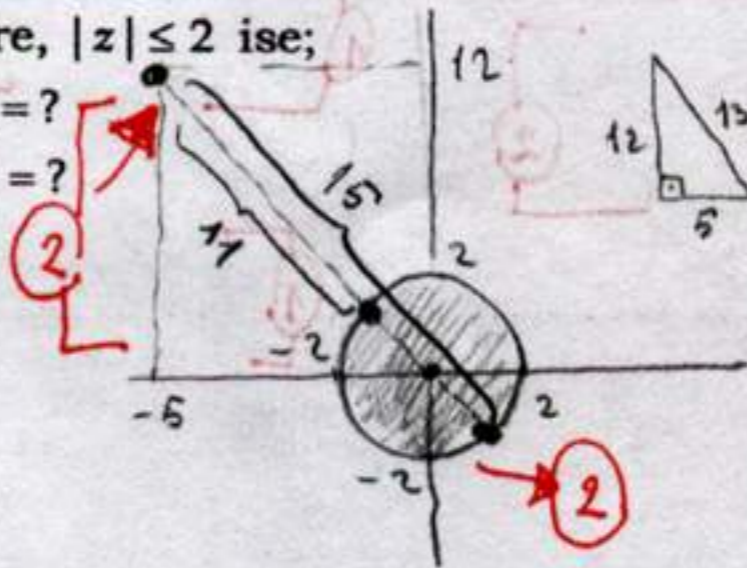
3

 $z \in \mathbb{C}$ olmak üzere, $|z| \leq 2$ ise;

$$\min(|z + 5 - 12i|) = ?$$

$$\max(|z + 5 - 12i|) = ?$$

10 puan



$$|z - (-5 + 12i)| : z \text{'in } "-5 + 12i", \text{ 'ye } 11 \pm 2 \text{ klığı}$$

$$\min(|z - (-5 + 12i)|) = 13 - 2 = 11$$

$$\max(|z - (-5 + 12i)|) = 13 + 2 = 15$$

4

$$z = \frac{\sqrt{3}}{2} + \frac{i}{2} \rightarrow \left(\frac{z}{z} \right)^{-3i} = ?$$

$$z = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$$

$$z = \text{cis } \frac{\pi}{6}$$

$$\bar{z} = \text{cis } -\frac{\pi}{6} = \text{cis } \frac{11\pi}{6}$$

$$= \left(\text{cis } \frac{11\pi}{6} \right)^{-3i}$$

$$= \left(e^{i \cdot 11\pi/6} \right)^{-3i}$$

$$= e^{-i \cdot 3i \cdot \frac{11\pi}{6}}$$

$$= e^{\frac{11\pi}{2}}$$

$$= e^{\frac{11\pi}{2}} + k \cdot 2\pi \quad (k \in \mathbb{Z})$$

5

 $e^x + 6 \cdot e^{-x} = 5$ denkleminin kökler toplamını bulun.

$$e^x + \frac{6}{e^x} = 5$$

$$e^{2x} + 6 = 5e^x$$

$$e^{2x} - 5e^x + 6 = 0$$

$$e^x \quad -3$$

$$e^x \quad -2$$

$$(e^x - 3) \cdot (e^x - 2) = 0$$

$$e^x - 3 = 0 \rightarrow e^x = 3 \rightarrow x_1 = \log_e 3 = \ln 3$$

$$e^x - 2 = 0 \rightarrow e^x = 2 \rightarrow x_2 = \log_e 2 = \ln 2$$

$$x_1 + x_2 = \ln 3 + \ln 2 = \ln 3 \cdot 2 = \ln 6$$

6

$$f(x) = 2^{\frac{x}{x+1}}$$

f'nin tersi de bir fonksiyon olduğuna göre, $f^{-1}(8) = ?$

$$y = 2^{\frac{x}{x+1}}$$

$$x = 2^{\frac{y}{y+1}}$$

$$\log_2 x = \frac{y}{y+1}$$

$$y \cdot \log_2 x + \log_2 x = y$$

$$\log_2 x = y(1 - \log_2 x)$$

$$y = \frac{\log_2 x}{1 - \log_2 x} = f^{-1}(x)$$

$$f^{-1}(8) = \frac{\log_2 8}{1 - \log_2 8} = \frac{\log_2 2^3}{1 - \log_2 2^3} = \frac{3 \log_2 2}{1 - 3 \log_2 2} = \frac{3}{1-3} = -\frac{3}{2}$$

10 puan

7

$$\log_3(80!) = a \rightarrow \log_9\left(\frac{1}{81!}\right) = ?$$

$$\rightarrow = \log_{3^2} (81!)^{-1}$$

$$= -\frac{1}{2} \log_3 81!$$

$$= -\frac{1}{2} [\log_3 81 \cdot 80!]$$

$$= -\frac{1}{2} [\log_3 81 + \log_3 80!]$$

$$= -\frac{1}{2} [\log_3 3^4 + a]$$

$$= -\frac{1}{2} [4 \log_3 3 + a] = -\frac{1}{2} (4+a)$$

10 puan

8

$$\log_a b + \log_b a = \frac{5}{2} \rightarrow \frac{a^2 + b^2}{b} = ?$$

$$\log_a b + \frac{1}{\log_a b} = \frac{5}{2}$$

$$(\log_a b)^2 + 1 = \frac{5 \log_a b}{2}$$

$\log_a b = m$ olsun

$$m^2 + 1 = \frac{5m}{2}$$

$$2m^2 + 2 = 5m$$

$$2m^2 - 5m + 2 = 0$$

$$\begin{matrix} 2m & -1 \\ m & -2 \end{matrix}$$

$$(2m-1) \cdot (m-2) = 0$$

$$m = \frac{1}{2} \quad m = 2$$

$$\log_a b = \frac{1}{2}$$

$$b = a^{1/2}$$

$$b^2 = a$$

$$\wedge \log_a b = 2$$

$$b = a^2$$

$$1+1 = 2$$

10 puan

9

$f(x) = \log\left(\frac{x^2-x-20}{x-2}\right)$ Verilen fonksiyonun en geniş tanım kümesini bulun.

$$\frac{x^2-x-20}{x-2} > 0$$

$$\frac{(x-5) \cdot (x+4)}{x-2} > 0$$

$$x=5 \vee x=-4$$

$$x=2 \text{ lain tanımsız}$$

| | | | | | |
|------------|-----------|----|---|---|-----------|
| x | $-\infty$ | -4 | 2 | 5 | $+\infty$ |
| x^2-x-20 | + | 0 | - | - | 0 |
| $x-2$ | - | - | 0 | + | + |
| f(x) | | | | | |

$$(-4, -2) \cup (5, +\infty)$$

10 puan

10

$x \in \mathbb{Z}^+$ olmak üzere $f(x) = \log\left(\frac{x}{1+x}\right) \rightarrow f(1)+f(2)+f(3)+\dots+f(99) = ?$

$$= \log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \dots + \log \frac{99}{100}$$

$$= \log \left(\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \dots \cdot \frac{99}{100} \right) = \log \frac{1}{100} = \log 10^{-2} = -2 \log 10$$

10 puan



Başarılar...
Ümit CANLI